

Changing Opinions in a Changing World: a New Perspective in Sociophysics

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We propose a new model of opinion formation, the *Opinion Changing Rate* (OCR) model. Instead of investigating the conditions that allow consensus in a world of agents with different opinions, we study under which conditions a group of agents with a different natural tendency (*rate*) to change opinion can find agreement. The OCR is a modified version of the Kuramoto model, one of the simplest models for synchronization in biological systems, here adapted to a social context. By means of several numerical simulations we illustrate the richness of the OCR model dynamics and its social implications.

Keywords: Sociophysics; Opinion Dynamics Models; Synchronization

1. Introduction

The world changes and we change with it. But everyone in a different way. There are conservative people that strongly tend to maintain their opinion or their style of life against everything and everyone. There are more flexible people that change ideas very easily and follow the current fashions and trends. Finally, there are those who run faster than the rest of the world anticipating the others. These different tendencies can be interpreted as a continuous spectrum of different degrees of natural inclination to changes. By changes here we mean change of opinions or more in general change of ideas, habits, style of life or way of thinking.

The last years have seen a large interest in the physics community towards the description and modeling of social systems^{1,2,3}. In particular, Monte Carlo simula-

tions have become an important part of *sociophysics*, enlarging the field of interdisciplinary applications of statistical physics. Many of the most popular sociophysics models⁵ deal with opinion dynamics and consensus formation but have the limitation of not taking into account the individual inclination to change, a peculiar feature of any social system. In fact, as Charles Darwin wrote, “it is not the strongest that survives, nor the most intelligent; it is the one that is most adaptable to change”.

In this paper we show how such an individual inclination to change, differently distributed in a group of people, can affect the opinion dynamics of the group itself. The first advantage of adopting this new perspective is the possibility of using the knowledge accumulated in the study of coupled oscillators. If we switch from the question “*Could agents with initial different opinions reach a final agreement?*” into the more realistic one “*Could agents with a different natural tendency to change opinion reach a final agreement?*”, we introduce the concept of natural opinion changing rate, that is very similar to the characteristic frequency of an oscillator. In such a way, we can treat consensus as a peculiar kind of synchronization (frequency locking) which has been very well studied in different contexts by means of the Kuramoto model⁶. The paper is organized as follows. In Section 2 we briefly review the Kuramoto model, a system of ordinary differential equations introduced to model synchronization in biological coupled oscillators. In Section 3 we propose the Opinion Changing Rate (OCR) model, a new model of opinion dynamics inspired by the Kuramoto model. We discuss in detail the definitions, and we explore the rich dynamics of the model by means of extensive molecular dynamics simulations. In Section 4 we draw the conclusions and we outline possible generalizations and future developments.

2. The Kuramoto model

Coupled oscillators are ubiquitous in nature^{7,8}. A large number of biological, physical and chemical systems can be thought of as a large ensemble of weakly interacting nonidentical oscillators. Examples include flashing fireflies⁹, and chorusing crickets^{10,11}, pacemaker cells in the heart¹² and in the brain, coupled laser arrays⁸. One interesting phenomenon common to all these systems is their ability to collectively synchronize: a large number of the oscillators lock onto a common frequency despite the difference in their natural frequencies. The most striking visual example is given by the synchronous flashing of fireflies observed in summer season in some region of South Asia. At night thousands of fireflies meet on the trees along the river. Suddenly they start to emit flashes of light. Initially they act incoherently and after a short time they all flash in unison, creating one of the most beautiful effects of synchronization¹³.

Researches on synchronization aim at understanding what are the basic mechanisms responsible for the collective synchronous behavior in a given population. The Kuramoto model of coupled oscillators is one of the simplest and most suc-

successful models for synchronization. It is simple enough to be analytically solvable, still retaining the basic principles to produce a rich variety of dynamical regimes and synchronization patterns. It was originally proposed in 1975 by Kuramoto⁶ as an analytically tractable version of the Winfree's mean-field model for large populations of biological oscillators¹⁴. Since then the Kuramoto model has been analyzed more deeply and several extensions and generalizations have been considered^{15,16,17,7,8,18}. It has also been linked to several physical problems, including Landau damping in plasmas¹⁶ and the dynamics of Josephson junction arrays¹⁹. The Kuramoto model describes a population of N periodic oscillators having natural frequencies ω_i and coupled through the sine of their phase differences. The dynamics of the model is given by

$$\dot{\theta}_i(t) = \omega_i + \frac{K}{N} \sum_{j=1}^N \sin(\theta_j - \theta_i) \quad i = 1, \dots, N \quad (1)$$

where $\theta_i(t)$ is the phase (angle) of the i th oscillator at time t , while ω_i is its intrinsic frequency randomly drawn from a symmetric, unimodal distribution $g(\omega)$ with a first moment ω_0 (typically a Gaussian distribution or a uniform one). These natural frequencies ω_i are time-independent. The sum in the above equation is running over all the oscillators so that this is an example of a globally coupled system. The parameter $K \geq 0$ measures the coupling strength in the global coupling term. For small values of K , each oscillator tends to run independently with its own frequency, while for large values of K , the coupling tends to synchronize (in phase and frequency) the oscillator with all the others. Notice that the equations (1) can be transformed into an equivalent system of phase oscillators with a zero mean frequency, by a suitable transformation $\theta_i \rightarrow \theta_i - \Omega t$ to a rotating frame with a velocity $\Omega = \omega_0$.

In a beautiful analysis, Kuramoto showed that the model, despite the difference in the natural frequencies of the oscillators, exhibits a spontaneous transition from incoherence to collective synchronization, as the coupling strength is increased beyond a certain threshold K_c ⁶. Kuramoto studied the system in terms of a complex mean field order parameter defined as

$$r e^{i\Psi} = \frac{1}{N} \sum_{j=1}^N e^{i\theta_j} \quad , \quad (2)$$

where the magnitude $0 \leq r(t) \leq 1$ is a measure of the coherence of the population and $\Psi(t)$ is the average phase. The system of equations (1) can be rewritten, in terms of the mean field variable r and Ψ , as

$$\dot{\theta}_i(t) = \omega_i + Kr \sin(\Psi - \theta_i) \quad i = 1, \dots, N \quad . \quad (3)$$

In the limit of infinitely many oscillators, the system can be described by a contin-

uous probability density $\rho(\theta, \omega, t)$, satisfying the normalization condition

$$\int_0^{2\pi} \rho(\theta, \omega, t) d\theta = 1 \quad (4)$$

and the order parameter of Eq. (2) can be expressed in terms of $\rho(\theta, \omega, t)$ as:

$$r e^{i\Psi} = \int_0^{2\pi} \int_{-\infty}^{+\infty} e^{i\theta} \rho(\theta, \omega, t) g(\omega) d\theta d\omega \quad (5)$$

The dynamics of the system in terms of the probability density is governed by the continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial \theta} \{ [w + Kr \sin(\Psi - \theta)] \rho \} = 0 \quad (6)$$

since each oscillator moves with an angular velocity (frequency) $v_i = \dot{\theta}_i$ given by Eq.(3).

The main conclusions that can be drawn from a simple analysis of the previous formulas and from the results of numerical simulations are the following.

- In the limit $K \rightarrow 0$, equations (3) give $\theta_i(t) \approx \omega_i t + \theta_i(0) \forall i = 1, \dots, N$, and consequently the system is in a *phase incoherent state*. The individual phase variables $\theta_i(t)$ are not correlated and the average of $e^{i\theta_i(t)}$ over all oscillators, the sum on the right hand side of Eq. (2), is expected to go to zero with the system size as $N^{-1/2}$, yielding, for $N \rightarrow \infty$ and $t \rightarrow \infty$, a vanishing saturation value of the order parameter, i.e. $r_\infty \rightarrow 0$.
- On the other hand, in the strong coupling limit $K \rightarrow \infty$, Eq. (6) has a stationary solution in which all the oscillators have almost the same phase $\theta_i(t) \approx \Psi(t) \forall i$. Consequently $r_\infty \approx 1$ and we have a *global synchronization*. It is important to stress, however, that since Eq.(1) implicates the impossibility of simultaneously synchronize both phase and frequency, here synchronization means that all the oscillators have, for $t \rightarrow \infty$, *exactly* the same long term frequency (frequency locking) and *almost* the same phase. In fact there is a small phase shift between the oscillators and such phase shift is conserved in time ^{7,20}.
- In the intermediate situation of finite coupling a less degree of synchronization with $0 \leq r_\infty \leq 1$ is observed (*partial synchronization*). The stationary solution of Eq.(6) is a *phase coherent state* in which part of the oscillators clump around the mean phase $\Psi(t)$, while the others move freely out of synchrony. In fact, the typical oscillator is running with angular velocity $v_i = \omega_i + Kr \sin(\Psi - \theta_i)$. The oscillators in the coherent population will become stable locked at an angle such that $\omega_i = Kr \sin(\theta_i - \Psi)$ in the frame of reference in which the common velocity is set equal to zero. Such a condition

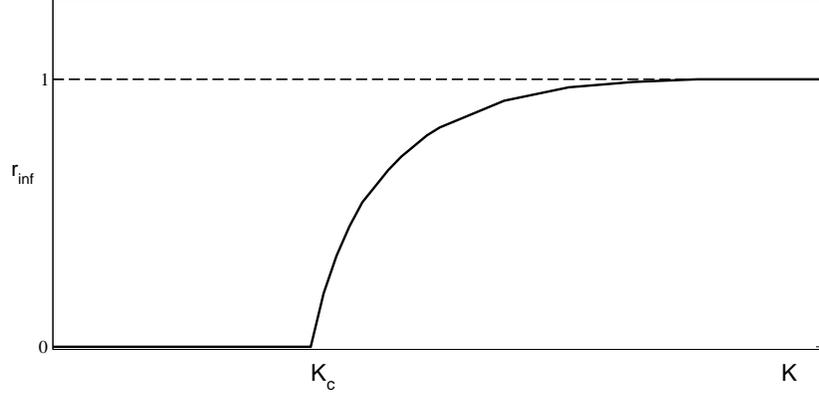


Fig. 1. Asymptotic order parameter r_∞ as a function of the coupling in the Kuramoto model

cannot be satisfied by the oscillators with $|\omega| \geq Kr$: these oscillators cannot be locked and their stationary density obeys $v\rho = \text{constant}$ according to Eq. (6).

The appearance of the coherent state happens at a critical coupling strength K_c .

Summing up, as N approaches infinity, the magnitude r_∞ of the complex mean field after a transient time should be zero in the incoherent state with $K \leq K_c$ and different from zero in the coherent state with $K > K_c$. Actually, as K increases beyond K_c , more and more oscillators will be recruited toward the mean phase $\Psi(t)$ and r_∞ is expected to continuously increase from zero to one (see Fig.1).

The details of this dynamical phase transition for the globally coupled periodic oscillators have been studied both analytically and numerically. For instance, a detailed description of the Kuramoto's analytical arguments to study the bifurcation from the incoherence state can be found in Ref. ⁷. Here we only summarize the main results obtained. The critical value K_c can be expressed as a function of $g(\omega)$, namely $K_c = 2/(\pi g(0))$. For instance in the simple case of a Lorentzian distribution $g(\omega) = (\gamma/\pi)/(\gamma^2 + \omega^2)$ it results $K_c = 2\gamma$ and it is also possible to obtain an analytical expression for the magnitude of the order parameter, $r = \sqrt{1 - K_c/K}$, with $K > K_c$.

3. The "Opinion Changing Rate" Model

Usually, consensus models ^{21,22,23} in sociophysics deal with N individuals (or agents): each individual $i = 1, \dots, N$ is characterized by an opinion x_i and is in interaction with all the others. The opinions are integer numbers (for instance +1 or -1) in the Sznajd ²¹ model, or real numbers in the range [0,1] in the Deffuant et al. ²² and

in the Hegselmann and Krause²³ model. The interaction dynamics of such models is governed by very simple deterministic rules. For instance, in the Sznajd model a pair of neighbouring agents on a square lattice convinces its six neighbours of the pair opinion if and only if the two agents of the pair share the same opinion. In the Deffuant et al. model a pair of agents i and k is selected at random. If the opinions x_i and x_k differ by more than a fixed parameter ϵ nothing happens because the two agents think too differently to interact and eventually find an agreement. If $|x_i - x_k| < \epsilon$ then both opinions get closer to each other by an amount $\mu \cdot |x_i - x_k|$, tuned by the parameter μ . The main goal of the opinion dynamics models is to figure out whether and when a complete or partial consensus can emerge out of initially different opinions, no matter how long it takes for the consensus to be reached. For instance the Deffuant et al. model presents a rich scenario of outcomes as the two control parameters ϵ and μ are varied.

The first difference between such models and our approach is that we are interested

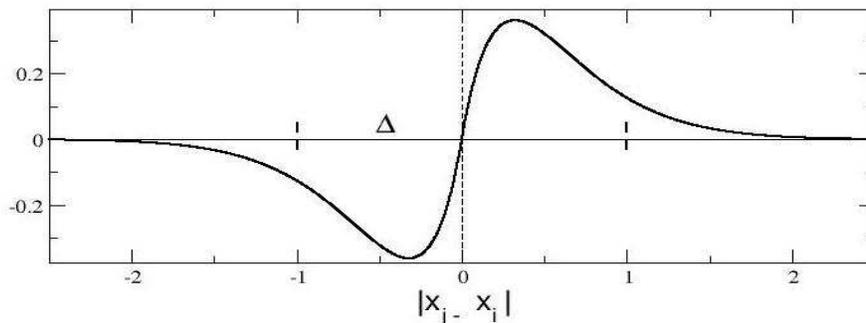


Fig. 2. Behavior of the coupling term in Eq.(7), namely $\sin(x_j - x_i)e^{-\alpha|x_j - x_i|}$, as a function of the reciprocal distance between two opinions. We set $\alpha = 3$ and we consider a distribution of the initial individual opinions $x_i(t = 0)$ in the range $[-\Delta, \Delta]$, with $\Delta = 1$, in order to ensure that, at the beginning of the simulation, the interaction between each couple of agents is not negligible.

mainly on the dynamical aspects of the consensus formation and not only on the equilibrium ones. Of course, at variance with the phases in the Kuramoto model, in our model we do not want periodic opinions nor limited ones. In fact, here opinions have a very general meaning and can represent the style of life, the way of thinking or of dressing etc. Then we do not consider periodic boundary conditions and assume that $x_i \in]-\infty + \infty[\forall i = 1, \dots, N$. Thus two opinions can diverge in the time evolution. Furthermore, it is quite natural that individuals with very different opinions will tend to interact less. Such an idea is present in the approach by Deffuant et al. where it is taken into account by means of the parameter ϵ . In our model we assume, instead, that the coupling between two agents is a decreasing function of their opinion difference and should vanish when their opinions are very different. Finally, as stressed in the introduction, in our model we are not so much

interested in the particular opinion of each agent: rather, we want to see under what conditions it is possible to find agreement among agents with a different velocity (*rate*) in changing their opinion. Thus, the name of the model: the *Opinion Changing Rate* (OCR) model. Taking into account all the previous requirements, we have adopted the following dynamics for the OCR model:

$$\dot{x}_i(t) = \omega_i + \frac{K}{N} \sum_{j=1}^N \sin(x_j - x_i) e^{-\alpha|x_j - x_i|} \quad i = 1, \dots, N \quad (7)$$

Here $x_i(t)$ is the opinion (an unlimited real number) of the i th individual at time t , while ω_i represents the so called *natural opinion changing rate*, i.e. the intrinsic inclination, or natural tendency, of each individual to change his opinion (corresponding to the *natural frequency* of each oscillator in the Kuramoto model). As in the Kuramoto model, also in the OCR model the ω 's are randomly drawn from a given symmetric, unimodal distribution $g(\omega)$ with a first moment ω_0 . Usually a uniform or a Gaussian distribution centered at ω_0 is used. In this way we simulate the fact that in a population there are: 1) conservative individuals, that naturally tend to change their opinion very slowly, and thus are characterized by a value of ω_i smaller than ω_0 ; 2) more flexible people, with $\omega_i \sim \omega_0$, that change idea more easily and follow the new fashions and trends; 3) individuals with a value of ω_i higher than ω_0 , that tend to anticipate the others with new ideas and insights. In the equation (7) K , as usual, is the coupling strength. The exponential factor in the coupling term ensures that, for reciprocal distance higher than a certain threshold, tuned by the parameter α , opinions will no more influence each other, see Fig.2 . In the following, without loss of generality, we fix $\alpha = 3$. We also notice that, because the natural frequencies ω_i remain always constant in time, in absence of interaction (i.e. for $K=0$) each opinion would change at a constant rate, equal to its associated natural rate, independently from the other opinions.

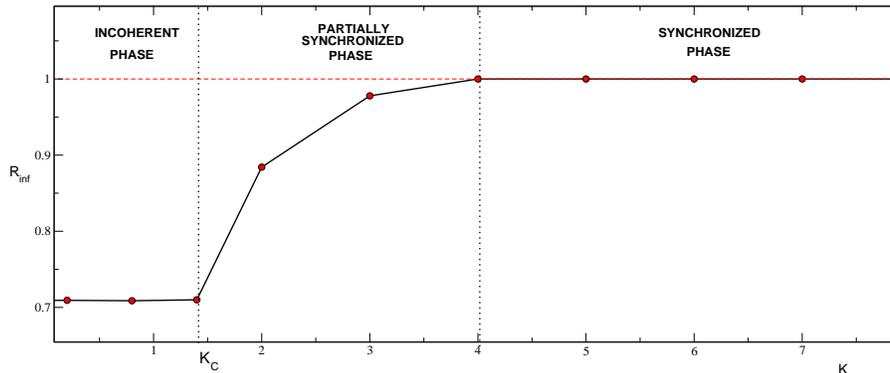


Fig. 3. Asymptotic order parameter R_∞ as a function of the coupling K in the OCR model. Simulations have been performed for $N=1000$ agents and $\alpha = 3$ (see text).

The aim of this paper is to study the opinion dynamics of the OCR model by solving numerically the set of ordinary differential equations (7) for a given distribution of the ω 's (natural opinion changing rates) and for a given coupling strength K . In particular, we want to find out if, as a function of K , there is a transition from an incoherent phase, in which people tends to have opinions changing at different velocity, i.e. at different 'opinion changing rate', to a synchronized one in which all the people change opinion with the same velocity. The latter, in this case, will represent the common trend of the entire society, or, in other words, the common rate of the '*changing world*'.

Because of the non-periodicity of the opinion values in our model, we cannot use the Kuramoto mean field order parameter of Eq.(3) to measure the degree of synchronization. On the other hand there are a series of different alternatives. Here we adopt an order parameter related to the standard deviation of the opinion changing rate $\dot{x}_j(t)$. Such a parameter, indicated with $R(t)$, is defined as:

$$R(t) = 1 - \sqrt{\frac{1}{N} \sum_{j=1}^N (\dot{x}_j(t) - \dot{X}(t))^2} \quad (8)$$

where $\dot{X}(t)$ is the average over all individuals of $\dot{x}_j(t)$. From Eq.(8) it follows that:

- $R = 1$ in the fully synchronized phase, where all the agents have exactly the same opinion changing rate (and very similar opinions).
- $R < 1$ in the incoherent or partially synchronized phase, in which the agents have different opinion changing rates and different opinions.

3.1. Phase Transition

Our numerical simulations have been performed typically with $N=1000$ agents and with an uniform distribution of the initial individual opinions $x_i(t=0)$ in the range $[-\Delta, \Delta]$. Such a range has to be chosen consistently with the parameter α in the coupling term (see Fig.2), in order to ensure that at the beginning of the simulation all the agents would interact with each others. In the following we will set $\Delta = 1$. The natural opinion changing rates ω_i are taken from a uniform distribution in the range $[0,1]$ * We have checked that the alternative choice of a Gaussian distribution does not change significantly the results of the simulations. We fix the value of the coupling K and we let the system evolve by means of Eq.(7) until a stationary value for the order parameter $R_\infty = R(t = \infty)$ is obtained.

In Fig.3 we plot the asymptotic order parameter as a function of K for a system with $N = 1000$ agents. It is easy to recognize a Kuramoto-like transition from an

*As for the Kuramoto model, in the OCR model a uniform distribution of opinion changing rates in the range $[0, \omega_{max}]$ can always be transformed into the range $[0,1]$ by a scaling of the coupling constant $K' = K \cdot \omega_{max}$.

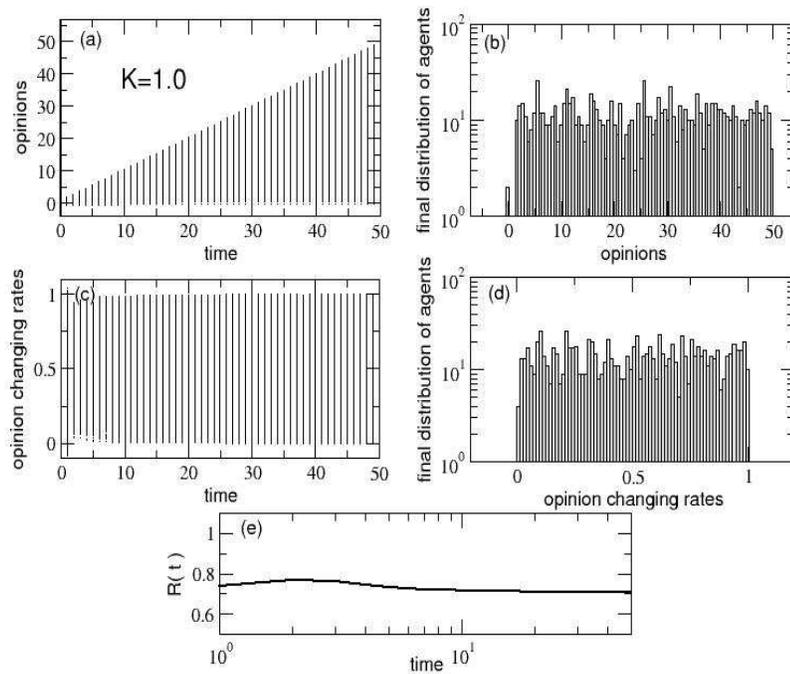


Fig. 4. OCR model for $N = 1000$ and $K = 1.0$. In panels (a) and (c) we plot the time evolution of the opinions and of the opinion changing rates, while in panels (b) and (d) we show the respective final distributions of agents. Notice that, in panels (a) and (c) we report, for each time step, 1000 points corresponding respectively to the opinions and opinion changing rates of the N agents. In panel (e), we report the time evolution of the order parameter R . Being in the incoherent phase, the system rapidly reaches an homogenous state for the opinions as well as for the frequencies, with a saturation value of the order parameter less than one.

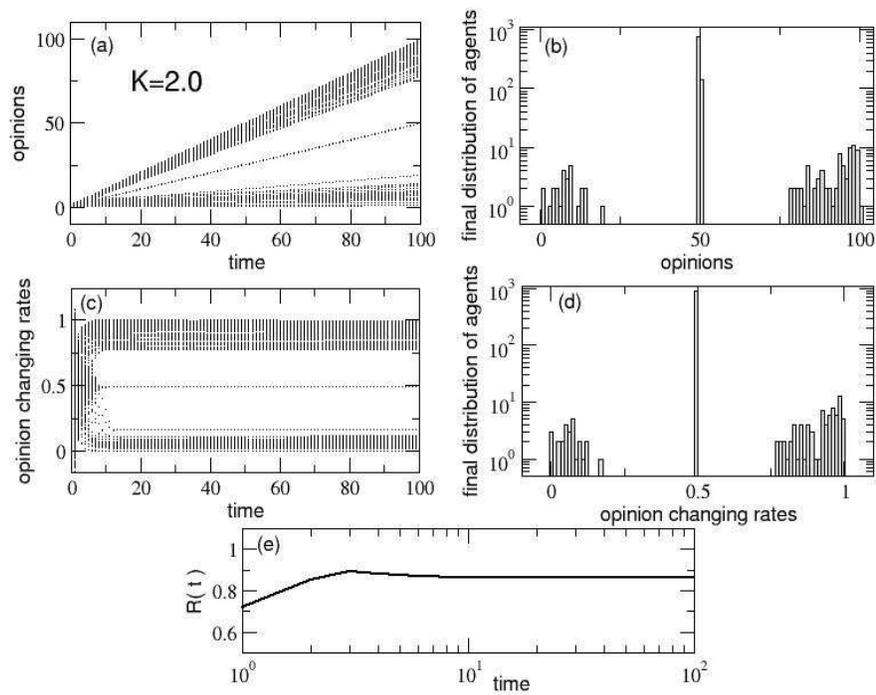


Fig. 5. OCR model for $N = 1000$ and $K = 2.0$. Same quantities as in Fig.4. In this case we are in the partially synchronized phase and the system splits into three clusters: a central one, with the largest part of the agents, and two lateral ones. The saturation value of the order parameter is still less than one, but it is larger than in the incoherent phase (previous figure).

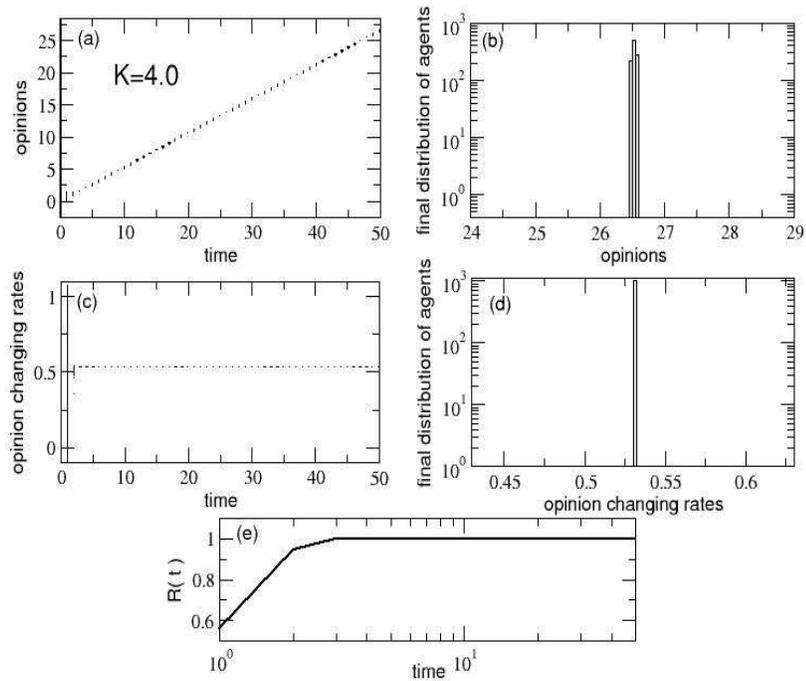


Fig. 6. OCR model for $N = 1000$ and $K = 4.0$. Same quantities as in Fig.4. In this case the coupling is strong enough so that all the agents reach a synchronized state (frequency locking). We observe a single final cluster, both in opinions and in opinion changing rates. The asymptotic order parameter rapidly becomes equal to one.

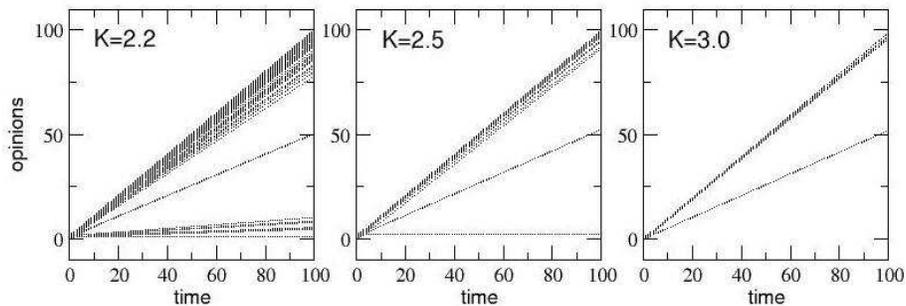


Fig. 7. Time evolution of opinions for three values of the coupling K in the partially synchronized phase. For values of K larger than 2.0 (compare with Fig.5), the 'conservative' group rapidly vanishes. The same thing happens to the 'progressist' group for K larger than 3.0

incoherent phase (for $K < K_c \sim 1.4$) to a partially coherent (for $K \in [1.4, 4.0]$) and, finally, to a fully synchronized phase (for $K > 4.0$). A better characterization of K_c as a function of N , α and Δ is under current study. We now focus on the details of the dynamical evolution in each of the three phases.

In Fig.4 we show the time evolution of the opinions and of the opinion changing rates (angular velocities or frequencies) with the respective final distributions for a small value of the coupling, namely $K = 1.0$. On the bottom part of the figure we plot the order parameter time evolution. Being in the incoherent phase, the system remains in a configuration of agents with finite standard deviation of opinion changing rates: because of the weak interaction with the others, each agent keeps his natural opinion changing rate and the different opinions will diverge in time without reaching any kind of consensus. We could also look at this as to an '*anarchical*' society. In such a phase the order parameter R takes the minimum possible value that, at variance with the Kuramoto's model, here is not zero.

In Fig.5 we show the case $K = 2.0$. The coupling is still weak but strong enough to give rise to three different clusters of evolving opinions, each with a characteristic changing rate: the largest number of the agents, representing "the world's way of thinking" or, if you want, the "public opinion", moves with an intermediate angular velocity, but there is a consistent group of people remaining behind them and also a group of innovative people (quicker in supply new ideas and ingenuity). From a political point of view, we could interpret this situation as a 'bipolarism' with a large number of 'centrists'. In this case the order parameter is larger than in the previous example, but still less than one because the opinion synchronization is only partial.

Finally, in Fig.6 we report the case $K = 4.0$. The coupling is so strong that all the opinions change at the same rate: in the histograms we observe a single final cluster, both in opinions and in opinion changing rates. It is important to stress, however, that, as in the Kuramoto model²⁰ also in the equations (7) of the OCR model is impossible to synchronize perfectly both phase and frequency. In fact, all the oscillators assume *exactly* the same frequency (frequency locking) and *almost* the same phase, with a small phase shift between them²⁰. In this '*dictatorial*' society all the agents think in the same way and follow the same trends and fashions. Although the *natural* frequencies of the agents are - as in the previous examples - different from each others, their opinion changing rates rapidly synchronize (frequency locking) and thus the order parameter R reaches a saturation value equal to one.

It is interesting to notice that, by increasing the value of K from 2.0 to 4.0 (see Fig.7), the group with a low opinion changing rate rapidly disappears, leaving place only to two groups, the central one and the innovative (fast) one. The survival of the fast group is not fully understood although it seems to support the Darwin's statement reported in the introduction.

Above $K = 3.0$ also the fast group vanishes until, near $K = 4.0$, only the central synchronized group remains. Some conclusions, in agreement with the intuition, can

be drawn from the numerical simulations we have performed. In the OCR model, in order to ensure a 'bipolarism', i.e. an equilibrium between the conservative and progressist components, a changing society needs a level of coupling K strictly included in a narrow window ($1.5 < K < 2.5$) inside the partially synchronized phase. Otherwise such an equilibrium will be broken and the final result will be anarchy or dictatorial regime.

3.2. Metastability

In the previous subsection we have shown under what conditions the OCR model exhibits dynamical synchronization in the agents' frequency and opinion. In this subsection we investigate the dynamics of the OCR model in the case in which the initial opinions are synchronized from the beginning. Instead of extracting the initial opinions from a uniform distribution with halfwidth $\Delta = 1$, as in the previous simulations, here we start all the agents with the same opinion ($\Delta = 0$). The distribution of the natural inclinations to change, $g(\omega)$, is the same as considered in subsection 3.1. In Fig.8 we report the results of the numerical simulation for

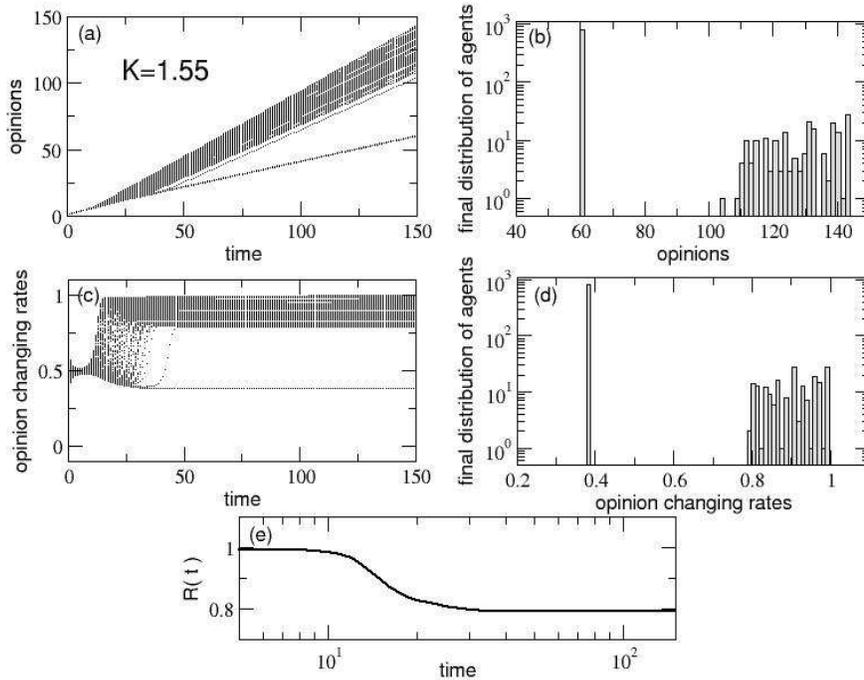


Fig. 8. OCR model for $N=1000$, $K=1.55$ and completely synchronized initial opinions. The system remains synchronized for a short time and then relaxes to a stationary state where, apart from the main synchronized group, a largely spread cluster of progressists appears.

$N=1000$ and $K = 1.55$, a value of the coupling constant higher than the critical value $K_c = 1.4$. We observe that the opinion changing rates initially try to synchronize in only one big cluster, and all the agents have very similar (but not identical) opinions. Then, rapidly and progressively, many agents leave the (almost) common opinion and increase their opinion changing rates, concentrating in a new, largely spread, cluster of progressists. Also in this case, the conservative group observed in Fig.5 does not appear. Correspondingly, the order parameter R stays for a while in a metastable state around the value $R = 1$, then rapidly decreases and stabilizes at $R = 0.8$.

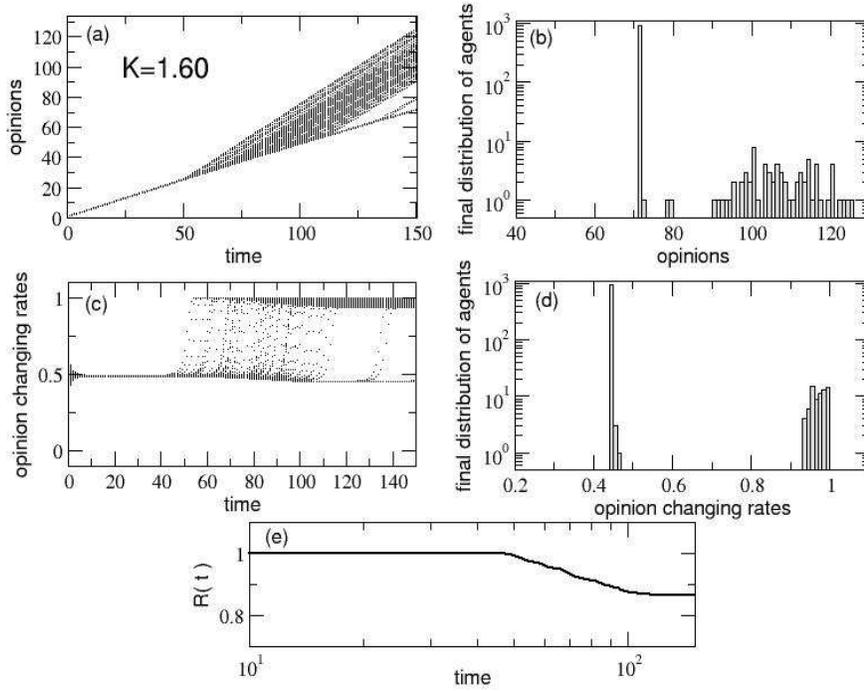


Fig. 9. OCR model for $N = 1000$, $K = 1.6$ and completely synchronized initial opinions. Same plot as in Fig.8 The initial synchronized state behaves as a metastable state whose life-time diverge approaching the value of the coupling $K \sim 1.62$

In Fig.9 we report the results obtained for a larger value of K , namely $K = 1.6$. Here the life-time of the metastable state with $R = 1$ becomes visibly longer. We have checked that, approaching $K \sim 1.62$, the life-time of such a metastable state diverges exponentially. The presence of metastable states with a diverging life-time for values of the control parameter above the transition from the homogeneous to the non-homogeneous phase is a characteristics observed in a large class of systems²⁴. It seems to be related to a glassy-like dynamical behavior that would hinder syn-

chronization^{25,26}. The social consequence of this out-of-equilibrium scenario could be the metastability of a dictatorial regime (chosen as initial condition) approaching a critical value of interconnections between individuals.

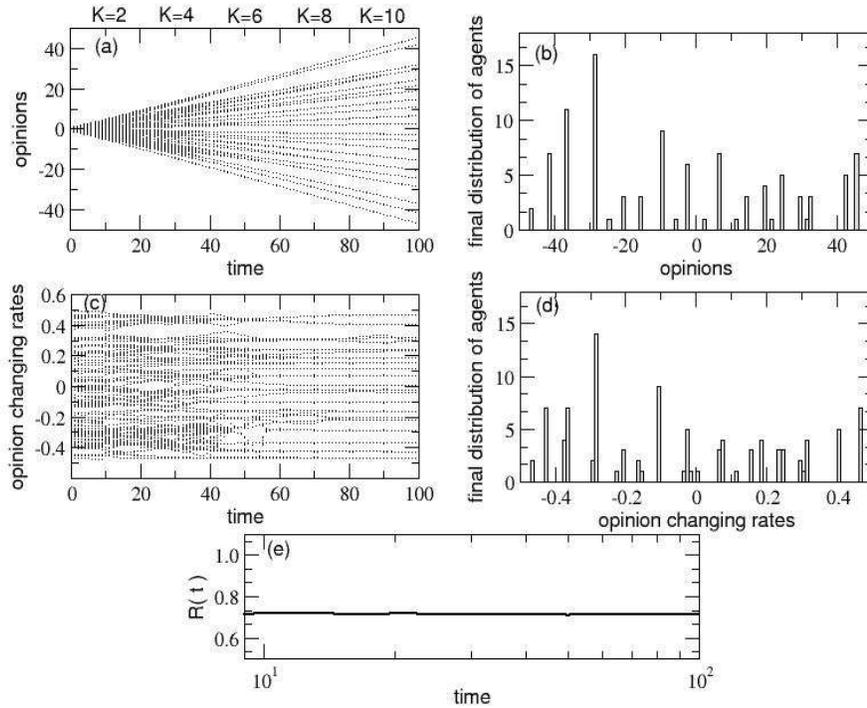


Fig. 10. OCR model for $N = 100$ and a value of K increasing with a constant rate from 0.1 up to 10.1. We plot the same quantities as in the previous figures

3.3. Variable Coupling

Finally we want to see what happens if the coupling K is let, respectively, to increase or decrease its value during the dynamics. In this way we can simulate a society in which the interconnections between the agents changes in time. The initial opinions and the natural changing rates are, as usual, uniformly distributed. At variance with the previous simulations, and only for a better visualization of the outputs, here we consider $N = 100$ and the natural opinion changing rates distributed in the interval $[-0.5, 0.5]$.

In Fig.10 we consider a case in which the coupling is uniformly increased from $K = 0.1$ to $K = 10.1$. The agents' opinions initially spread freely, and then rapidly freeze in a large number of non-interacting clusters with different changing rates and variable sizes. Note that the final distribution of clusters is different from those

in Fig.4, even if also in this case the order parameter R reaches the same minimum value, since the non-interacting clusters are uniformly distributed. The particular cluster distribution observed in Fig.10 cannot be obtained in simulations with a constant coupling. This could suggest that the increase of interactions between the members of a society (due to the improvement in transport and communications) would be determinant in order to stabilize that plurality of different (and consistent) clusters of opinions (ideologies, political parties, etc.) necessary for a democracy.

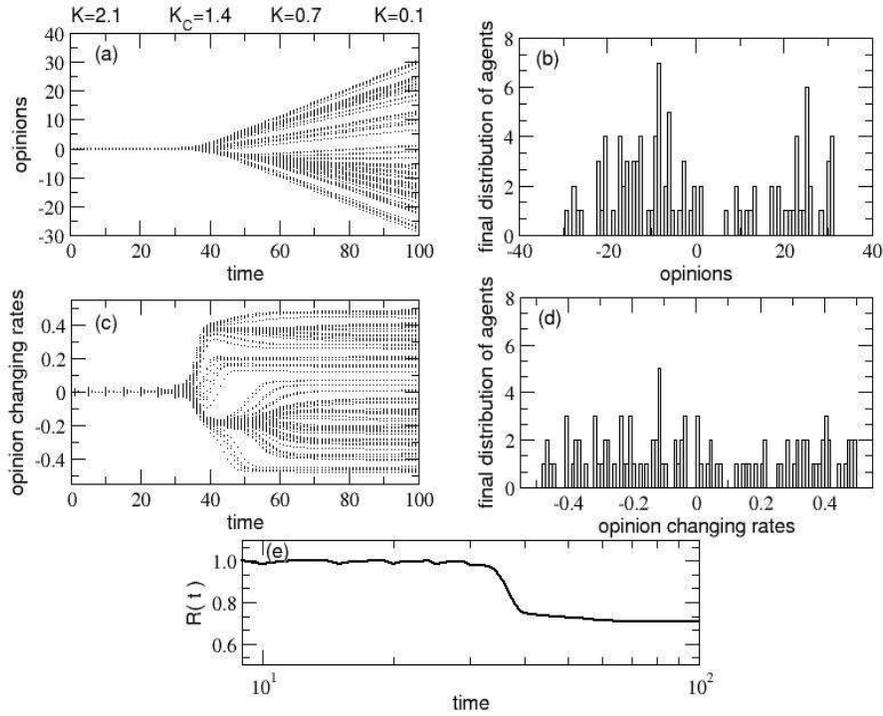


Fig. 11. OCR model for $N = 100$ and a value of K decreasing with a constant rate from 2.1 to 0.1. We plot the same quantities as in the previous figures.

In Fig.11, we consider the case in which the coupling is uniformly decreased from $K = 2.1$ (a value in the partially synchronized phase) to $K = 0.1$ (incoherent phase). Also in this case, after a synchronized initial evolution, as soon as the coupling exceed the value K_C , the opinions spread. However they do not form the same clusters as in the previous simulation: they behave more incoherently showing a slight tendency to form two groups with opposite opinion changing rates. This is typical of the partially synchronized case from which the system started (see Fig.5). Finally, the order parameter, from the unitary value corresponding to the synchronized initial evolution, goes down to about the same value of the

previous case (Fig.10). This scenario reminds the typical situation of the fall of a dictatorship, which is usually followed by a chaotic regime (see for example the situation in Iraq after the Saddam Hussein capture). In alternative, it could be interpreted as the dissolution of an empire in many little countries (think of the dissolution of the Roman Empire or, more recently, to the dissolution of the Soviet Union).

4. Conclusions

In spite of its simplicity, the OCR model seems to capture some general features of the opinion formation process. The model is based on a set of coupled ordinary differential equations governing the rate of change of agents' opinions. In particular with the OCR model we have introduced the concept of '*opinion changing rate*', transforming the usual approach to opinion consensus modeling^{21,22,23} into a synchronization problem: "how can one synchronize the opinion of many agents having a different natural inclination to change idea?". The OCR model shows many interesting features with a given social meaning. First of all, the model exhibits a phase transition from an incoherent phase to a synchronized one at a well defined critical threshold K_c . We have observed three main scenarios as a function of the coupling K , which can be interpreted as the degree of interconnection and/or communication among individuals.

The incoherent state emerging for $K < K_c$ can be interpreted, in social terms, as anarchy in a society of many individuals with scarce or disrupted communications. In alternative, if we generalize the concept of agents to represent groups of individuals, for instance social communities, instead of single individuals, the state we obtain for $K < K_c$ can be interpreted as a world with many isolated and non interacting cultures. On the other hand, when $K \gg K_c$, i.e. in the fully synchronized phase, the model shows the appearance of a dictatorial regime, or a globalized world, where social and cultural differences are constrained into a single way of thinking, notwithstanding the different tendencies of each individual. Finally, we found that bipolarism and democracy are possible only in an intermediate regime, with $K \sim K_c$, corresponding to a partially synchronized phase. In the latter regime the role of the initial conditions and of the dynamics becomes particularly important. For instance, the model shows, as we approach a critical value of interconnections among individuals, the metastability of a dictatorial regime, chosen as initial condition.

We have also simulated various situations in which the coupling K is changing in time. In such cases we have observed a clustering of opinions (for K increasing in time) and the breaking of synchronization (for K decreasing in time), respectively corresponding to the coexistence of many political parties, and to the dissolution of an empire or a dictatorship.

Future developments include more analytical approaches to the OCR model and some generalizations in order to study opinion synchronization in more complex

situations, as for instance:

- social agents placed on a complex network^{27,28}. In this way one can investigate the influence of the topology on the opinion formation dynamics, with particular attention to the role of node centrality²⁹.
- the introduction of an external field representing the pressure of mass-media.
- the addition of disorder in the coupling among the agents.

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